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A Thermodynamically Complete Model for One-Dimensional Two-Phase Flows With Heat Exchange

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ABSTRACT

This publication reports on a compact presentation of a one-dimensional model for the description of two-phase flows with heat exchange. This model was proposed earlier in the form of a system of equations, containing the heat conduction and entropy conservation equations for each phase. In this report the system is formulated in a form that is consistent with the formulation of the preceding report of the author. The present system of equations uniformly describes the evolution of averaged variables and interphase fluctuation parameters. The correctness of the model has been established for convex equations of state and for proper compaction dynamics kinetics.

RELEASE LIMITATION

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APPROVED FOR PUBLIC RELEASE

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Executive Summary

Multi-phase flows are common in modern civilian and military environments. For example, multi-phase modelling is an inevitable procedure required for the design of modern internal combustion engines and when analysing internal ballistics problems. In the last decade the area of multi-phase modelling has rapidly expanded due to the development of novel warheads dealing with heterogeneous energetic mixtures, the emergence of multi-phase issues in terrorism threats such as contamination bombs, and an extensive use of multi-phase media and porous materials for blast mitigation.

Thus, two-phase modelling is an urgent requirement for the analysis of novel weapons and target responses. However, the nomenclature of existing two-phase models requires the use of a specific software framework that deals with a separate description of the two different phases complemented with the pre-selection of exchange terms and connecting governing equations.

The present report deals with a model presentation based on a description of a two-phase mixture as an averaged medium, using conservation laws for parameters of the mixture complemented by governing equations for the disequilibrium parameters (interphase fluctuation parameters). This formulation is believed to be most suitable for the implementation of a model within the existing hydrocode frameworks via the user-defined subroutine apparatus.

The present model describes behaviour of a two-phase heat conductive isotropic mixture with the ability to exchange mass, momentum, and energy between phases. The heat conduction is described within the extended thermodynamics approach (hyperbolic heat conduction) that makes the final system of equations of hyperbolic type. Fundamentals of the model have been published in [1], which are slightly modified in the present report and reduced to a compact form consistent with the formulation [2]. Conditions of hyperbolicity of the system of equations comprising the model have been established in the report, which are linked with the natural conditions of convexity for the internal energy dependencies for each phase. The correctness of the model has also been established by restrictions on the compaction dynamics kinetics.

1. Romenski E., Resnyansky A.D., and Toro E.F., Conservative hyperbolic model for compressible two-phase flow with different phase pressures and temperatures, Newton Institute Preprint Series, Preprint NI04034-NPA, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, 2004.

2. Resnyansky A.D. A Thermodynamically Complete Model for Simulation of One-Dimensional Multi-Phase Flows, DSTO report, DSTO-TR-1510, Edinburgh, Australia, 2003.

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Contents

1. Introduction

Multi-phase media are widely involved in modern civilian and military technology environments. Examples of the media are reactive mixtures in high-speed engines, a variety of energetic and filling constituents employed in modern weapons, gas-liquid aerosols and porous buffer materials used for blast mitigation. In particular, the heterogeneous energetic mixtures are widely used for arranging a tailored energy release in munitions such as delayed reaction/initiation, afterburn, etc. Simulation of the behaviour of such materials requires a description with models which are capable of calculating multi-phase flows. The two-phase models, consisting of the conservation laws for mass, momentum and energy for each phase are very popular. In that case, the conservation laws are formulated for partial characteristics (the mass-related characteristics for each phase with respect to the mixture volume, containing several phases) and interconnected by the exchange terms. The system of equations for both phases is complemented by a tabulated governing equation for the volume concentration of a phase (the compaction dynamics equation). This approach has certain drawbacks: i) it involves non-conservative terms associated with so-called 'interfacial' (or configurational) pressure (e.g., see the frequently cited Baer-Nunziato (BN) publication [3] for a model that was first formulated in [4]); and ii) it is impossible to implement the model in a hydrocode with the user-defined subroutine interface because the description involves descriptions for each phase and it is in contradiction with the typical hydrocode structure. Many attempts to consider a multi-phase medium as an averaged one have been made, including a classic monograph by Truesdell [5]. However, a closed thermodynamic formulation, resulting in an efficient practical realisation, had not been proposed at that time. A variety of models have been recently developed in several papers [3, 4, 6, 7, 8]. However, they are not formulated as conservation laws and these non-conservative formulations complicate analysis of thermodynamical correctness of the models.

The present work employs an approach, enabling us to derive equations in the form of conservation laws. One of the first realizations of this approach has been published in [9]. This allowed one to formulate equations for the averaged parameters and mass concentration, so the effective averaging parameter, which is involved in the consideration, is the mass concentration of a phase. Realization of this approach as a computer code [9] resulted in significant numerical difficulties associated with the mass exchange in the areas of high mass concentration due to actual involvement of partial characteristics in the calculation process. A formulation [7] should be mentioned which modified the BN model into a form that combines three conservation laws for the mixture with four independent equations out of the original BN model. Replacement of the partial characteristics by specific characteristics of the phases ('real' densities, pressures, etc) resulted in the parametrization (see, e.g., [10]) that claims that the mass concentration and density are not the only parameters involved in the complete description of the phases' state. This parametrization dictates that characterisation of the phases' state has to involve both the mass and volume fractions. The models employing this parametrization resulted in a successful model such as [11]. Reduction of the model for the one-dimensional multi-phase flows to a compact form has been conducted in [2].

The model [2, 11] considers a two-phase flow without heat exchange, by neglecting the temperature and entropy differences between the phases. However, the heat exchange appears to be critical in mixtures that contain phases which are either highly energetic or have highly contrasting properties and very different thermo-physical characteristics. A model that takes into account the heat exchange on the basis of hyperbolic heat conductivity was formulated in [1]. The equation derivation in the present report modifies the model in order to preserve the characteristic structure of the original systems when reducing it to the two-phase case. The outcome suggests a reduced form of the model for the one-dimensional case, and proposes a set of parameters allowing one to derive a compact form of the model in this case. This formulation helps us: i) to provide a thermodynamically correct model; ii) to solve a Riemann problem for a two-phase mixture with the help of a single-phase Riemann solver applied to each phase; and iii) to formulate jump conditions in a compact form. This formulation could also be convenient for generalisation of the model to multi-phase case.

2. Basic single-phase model

To start, we recall a heat conduction single-phase model [12] for the case of isotropic material with a spherical stress tensor presented by pressure only. The model is based on the principles of extended thermodynamics [13] and uses a generalisation of Fourier's heat conduction law to a non-stationary heat conduction hyperbolic equation. We slightly generalise the model by including the right-hand side source terms that characterise the external mass, momentum, and heat production (this generalisation will be useful further on, when the system is used for the development of a two-phase model). The combined system of equations for the description of mechanical and thermal response of a material takes the following form [12]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = m_0 ,$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = n_0 ,$$

$$\frac{\partial \rho q}{\partial t} + \frac{\partial (\rho q u + T)}{\partial x} = -\frac{\rho q}{\tau} ,$$

$$\frac{\partial \rho (e + u^2/2)}{\partial t} + \frac{\partial [\rho u(e + u^2/2) + pu + TJ]}{\partial x} = l_0 ,$$

(1)

here m_0 is the mass production rate, n_0 is the momentum production rate and l_0 is the energy production rate. Dependent thermodynamic variables are calculated as

$$p = \rho^2 e_\rho , \qquad T = e_s , \qquad J = e_q .$$

(2)

They are associated with the following thermodynamic identity

$$T\,ds = de + p\,dV - J\,dq$$

where internal energy $e = e(\rho, s, q)$ is a given function of the material state. Independent variables of the system (1) are ρ, s, q, and u, which are specific density, specific entropy, a thermal impulse variable and particle velocity, respectively. The function τ determines the characteristic time of 'thermal relaxation' (see [12]). The dependent variables are pressure p, temperature T, and an entropy flux variable J, which are calculated from the internal energy as stated in (2). It should be noted that the origin of the variable q comes from the heat flux entering Fourier's law, but in the present form it has quite a vague physical meaning with units in (s·m²·grad)/kg. In fact, its physical meaning is that $J = e_q$ (J is measured in W/(m²·grad)), where the actual heat flux is presented by the term $J \cdot T$.

Expanding the last equation of the system (1) with the use of the mass and momentum conservation laws, we have

$$\rho \frac{de}{dt} + p \frac{\partial u}{\partial x} + \frac{\partial TJ}{\partial x} = l_0 - \left(e - u^2\big/2\right)m_0 - un_0 \ ,$$

here $d/dt = \partial/\partial t + u\,\partial/\partial x$ is the particle derivative. Using the calculation rules for dependent variables and the heat flux equation of the system (1), we can derive the entropy production equation:

$$\frac{\partial \rho s}{\partial t} + \frac{\partial \rho su + J}{\partial x} = \frac{1}{T}\left(l_0 - m_0\mu' - un_0 + \varphi J\right), \tag{3}$$

where $\varphi = \rho q/\tau$ and $\mu' = \mu - u^2/2$ when

$$\mu = e + p\,/\,\rho - T\,s - q\,J \tag{4}$$

is the chemical potential. The equation (3) shows that J is really the entropy flux and the entropy production is non-negative (in the absence of external mass and momentum production) when $q \cdot J \geq 0$.

For the derivation of the system of equations in the form of conservation laws, one more consequence of the system (1) will be useful when we deal with the two-phase case. We note that a consequence of the momentum equation could be rewritten as

$$\frac{\partial u}{\partial t} + \frac{\partial u^2\big/2}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{n_0 - um_0}{\rho} \ . \tag{5}$$

This equation has been used in [1, 9, 11] in order to derive an equation for the velocity interphase fluctuations in a two-phase single-temperature non-heat-conducting medium. This was possible because the non-conservative term in the left-hand side of (5) could be reduced to a gradient of the chemical potential because the temperature variation between the phases was neglected. In the present case, using the derivation rules (2) for the

dependent variables, the differential of the chemical potential (4) can be assessed as follows

$$d\mu = de + \frac{dp}{\rho} - \frac{p}{\rho^2} d\rho - T ds - s dT - q dJ - J dq = \frac{dp}{\rho} - s dT - q dJ .$$
(6)

From (6), equation (5) can be rewritten in the following form

$$\frac{\partial u}{\partial t} + \frac{\partial u^2/2}{\partial x} + \frac{\partial \mu}{\partial x} + s \frac{\partial T}{\partial x} + q \frac{\partial J}{\partial x} = \frac{n_0 - u m_0}{\rho} .$$
(7)

It is clearly seen from (7) that for a single-temperature medium (when temperature and entropy are not changing between the phases) in the absence of heat conduction, the velocity difference allowed one to keep the chemical potential in the divergent form and eliminate the temperature and entropy flux gradients from the velocity difference equation. For the heat conductive medium with varying temperature between the phases the situation is more complex. However, it is possible to derive another conservation law using a consequence from the heat conduction equation of the system (1) rewritten for q and the entropy conservation law (3). Multiplying the heat conduction equation by ρs and the equation (3) by q, after their summation we have:

$$\frac{\partial \rho s q}{\partial t} + \frac{\partial \rho s q u}{\partial x} + s \frac{\partial T}{\partial x} + q \frac{\partial J}{\partial x} = \frac{q}{T}\left(l_0 - m_0 \mu' - u n_0 - s T m_0\right) + \varphi \frac{qJ - sT}{T} ,$$

and, subtracting this equation from (7), we obtain a required conservation law

$$\frac{\partial\left(u - \rho s q\right)}{\partial t} + \frac{\partial\left(u^2/2 - \rho s q u + \mu\right)}{\partial x} =$$
$$= \frac{n_0 - u m_0}{\rho} - \frac{q}{T}\left(l_0 - m_0 \mu' - u n_0 - s T m_0\right) - \varphi \frac{qJ - sT}{T} .$$
(8)

For the sake of convenience we call the difference $u - \rho s q$ a chemical flux due to the association between the variable change and the gradient of μ':

$$\frac{\partial \rho \eta}{\partial t} + \frac{\partial\left(\rho \eta u + \mu'\right)}{\partial x} = \frac{n_0 - u m_0}{\rho} - \frac{q}{T}\left(l_0 - m_0 \mu' - u n_0 - s T m_0\right) - \varphi \frac{qJ - sT}{T} ,$$

where $\rho \eta = u - \rho s q$.

The system of equations (1) complemented by the auxiliary equations (3) and (8) and closed by relationships (2) are all necessary to derive equations for a two-phase model.

3. Selection of parameters for a two-phase model

Let us denote the averaged density of a two-phase medium by $\rho = m/V$, here m is mass of a representative volume and V is the volume occupied by the mass. Similarly, we can define specific densities of the phases $\rho_1 = m_1/V_1$ and $\rho_2 = m_2/V_2$, where $m = m_1 + m_2$ and $V = V_1 + V_2$. Multi-phase theories usually deal with so-called partial densities, which relate the phase masses to the whole volume such as: $\rho'_1 = m_1/V$ and $\rho'_2 = m_2/V$. These partial characteristics are key ones because the conservation laws for each phase are actually formulated only for the partial characteristics. For the case of a media with phases, which are capable of exchanging mass, momentum, and heat, the conservation laws in the one-dimensional case take the following form for the first phase:

$$\frac{\partial \rho'_1}{\partial t} + \frac{\partial \rho'_1 u_1}{\partial x} = m_0 \ ,$$

$$\frac{\partial \rho'_1 u_1}{\partial t} + \frac{\partial \left(\rho'_1 u_1^2 + p'_1 \right)}{\partial x} = n_0 \ ,$$

$$\frac{\partial \rho'_1 q_1}{\partial t} + \frac{\partial \left(\rho'_1 q_1 u_1 + T_1 \right)}{\partial x} = -\frac{\rho'_1 q_1}{\tau_1} \ , \tag{9}$$

$$\frac{\partial \rho'_1 \left(e'_1 + u_1^2/2 \right)}{\partial t} + \frac{\partial \left[\rho'_1 u_1 \left(e'_1 + u_1^2/2 \right) + p'_1 u_1 + T_1 J_1 \right]}{\partial x} = l_0 \ .$$

and for the second phase:

$$\frac{\partial \rho'_2}{\partial t} + \frac{\partial \rho'_2 u_2}{\partial x} = -m_0 \ ,$$

$$\frac{\partial \rho'_2 u_2}{\partial t} + \frac{\partial \left(\rho'_2 u_2^2 + p'_2 \right)}{\partial x} = -n_0 \ ,$$

$$\frac{\partial \rho'_2 q_2}{\partial t} + \frac{\partial \left(\rho'_2 q_2 u_2 + T_2 \right)}{\partial x} = -\frac{\rho'_2 q_2}{\tau_2} \ , \tag{10}$$

$$\frac{\partial \rho'_2 \left(e'_2 + u_2^2/2 \right)}{\partial t} + \frac{\partial \left[\rho'_2 u_2 \left(e'_2 + u_2^2/2 \right) + p'_2 u_2 + T_2 J_2 \right]}{\partial x} = -l_0 \ .$$

Here m_0, n_0, and l_0 are the mass, momentum, and heat exchange rates, respectively; u_i (i=1,2) are velocities of the phases, p'_1 and p'_2 are partial pressures within the phases, q_i (i=1,2) are thermal impulse variables for the phases, and e'_1 and e'_2 are specific internal energies. The thermodynamic identity

$$T \, ds = de + p \, dV \ - J \, dq = de - p \, d\rho/\rho^2 \ - J \, dq \ , \tag{11}$$

where V is specific volume, being applied to each of the phases, enables us to calculate partial pressure, temperature, and the entropy flux similar to (2):

$$p_1' = (\rho_1')^2 \frac{\partial e_1'}{\partial \rho_1'}, \quad T_1 = \frac{\partial e_1'}{\partial s_1}, \quad J_1 = \frac{\partial e_1'}{\partial q_1},$$

$$p_2' = (\rho_2')^2 \frac{\partial e_2'}{\partial \rho_2'}, \quad T_2 = \frac{\partial e_2'}{\partial s_2}, \quad J_2 = \frac{\partial e_2'}{\partial q_2},$$

(12)

if dependencies of specific energy on ρ', s, and q are given:

$$e_1 = e_1'(\rho_1', s_1, q_1) , \quad e_2 = e_2'(\rho_2', s_2, q_2) .$$

(13)

It should be noted that the definition of partial pressure is based on the application of the thermodynamic identity (11) with respect to partial density. Thus, the traditional approach to the calculation of two-phase flows is to solve numerically systems (1) and (2), pre-selecting the exchange terms m_0, n_0 and l_0, and tabulating the 'equations of state' in the form (13) (for the sake of convenience, we call the relations such as (13) equations of state), using (12) for the calculation of pressure, temperature, and the entropy flux.

We can complement the systems (9-10) with additional conservation laws for the entropy flux similar to (3)

$$\frac{\partial \rho_1' s_1}{\partial t} + \frac{\partial \rho_1' s_1 u_1 + J_1}{\partial x} = \frac{1}{T_1}\left(l_0 - m_0\mu_1' - u_1 n_0 + \varphi_1 J_1\right) = R_1 ,$$

$$\frac{\partial \rho_2' s_2}{\partial t} + \frac{\partial \rho_2' s_2 u_2 + J_2}{\partial x} = \frac{1}{T_2}\left(-l_0 + m_0\mu_2' + u_2 n_0 + \varphi_2 J_2\right) = R_2 ,$$

(14)

and for the chemical flux as follows from (8):

$$\frac{\partial\left(u_1 - \rho_1' s_1 q_1\right)}{\partial t} + \frac{\partial\left(u_1^2/2 - \rho_1' s_1 q_1 u_1 + \mu_1\right)}{\partial x} =$$

$$= \frac{n_0 - u_1 m_0}{\rho_1'} - \frac{q_1}{T_1}\left(l_0 - m_0\mu_1' - u_1 n_0 - s_1 T_1 m_0\right) - \varphi_1 \frac{q_1 J_1 - s_1 T_1}{T_1} = N_1 ,$$

$$\frac{\partial\left(u_2 - \rho_2' s_2 q_2\right)}{\partial t} + \frac{\partial\left(u_2^2/2 - \rho_2' s_2 q_2 u_2 + \mu_2\right)}{\partial x} =$$

$$= -\frac{n_0 - u_2 m_0}{\rho_2'} + \frac{q_2}{T_2}\left(l_0 - m_0\mu_2' - u_2 n_0 - s_2 T_2 m_0\right) - \varphi_2 \frac{q_2 J_2 - s_2 T_2}{T_2} = N_2 .$$

(15)

It should be noted that the chemical potentials μ_i may involve in their definitions (that are identical to (4)) equally partial (ρ_i' and p_i') or 'real' (ρ_i and p_i) characteristics, because $p_i/\rho_i = p_i'/\rho_i'$ (see [2]).

The procedure of averaging, having been employed in [7], involves the introduction of the averaged density, pressure, and velocity. We introduce as in [10] the mass concentration of

the first phase as $c = c_1 = m_1/m$, then for the second phase $c_2 = m_2/m = 1 - c$. Then we can calculate the partial densities

$$\rho'_1 = m_1/V = (m_1/m)\cdot(m/V) = \rho c \quad, \qquad \rho'_2 = \rho(1 - c) \quad. \tag{16}$$

Introducing volume concentration for the first phase as $\theta_1 = \theta = V_1/V$ and for the second phase as $\theta_2 = 1 - \theta = V_2/V$, we can recalculate the specific densities of the phases as follows

$$\rho_1 = m_1/V_1 = (m_1/m)\cdot(m/V)\cdot(V/V_1) = \rho c/\theta \quad, \qquad \rho_2 = \rho(1 - c)/(1 - \theta) \ . \tag{17}$$

It is common (see [3, 4, 6-9, 11]) to tabulate a governing equation for the parameter θ (the compaction dynamics equation) evolving θ along the particle path of a mixture. Then from (15) an evolutionary equation for c in the form of the conservation law follows from the conservation of mass for the first or second phase. Thus, selection of the parameters related to density, namely, the averaged density and mass and volume fractions can be arranged in a natural way.

Let us calculate pressure in an averaged medium. Firstly, we link the local pressures and densities with the partial ones. We consider an alternative presentation to (13) for the equations of state in the following form

$$e_1 = e_1(\rho_1, s_1, q_1) \quad, \qquad e_2 = e_2(\rho_2, s_2, q_2) \quad. \tag{18}$$

Then, from (12) and (16) it follows that

$$p'_1 = (\rho'_1)^2 \frac{\partial e_1}{\partial \rho'_1} = (\rho_1\theta)^2 \left(\frac{1}{\theta} \cdot \frac{\partial e_1}{\partial \rho_1} \right) = \theta \cdot \rho_1^2 \frac{\partial e_1}{\partial \rho_1} = \theta \cdot p_1 \quad,$$

$$p'_2 = (1 - \theta)p_2 \ , \tag{19}$$

where the following denotations for the local pressures are used:

$$p_1 = \rho_1^2 \frac{\partial e_1}{\partial \rho_1} \quad, \quad p_2 = \rho_2^2 \frac{\partial e_2}{\partial \rho_2} . \tag{20}$$

Obviously, choice of the equation of state in the form e or e' does not affect the calculation rules (12) for temperature T and entropy flux J.

Selection of the parameters for the variables associated with momentum, thermal impulse, and entropy is not so easy. Because our objective is to derive equations in the form of conservation laws, it seems obvious to select parameters from combinations of the basic variables entering the conservation laws for the phases (9) and (10).

We start with the momentum conservation law that gives for the total momentum the combination $\rho'_1 u_1 + \rho'_2 u_2$. Alternative combinations resulting in conservation laws are either

$$\rho'_1 u_1 - \rho'_2 u_2 \tag{21}$$

or

$$u_1 - \rho'_1 s_1 q_1 - u_2 + \rho'_2 s_2 q_2 \; . \tag{22}$$

The last combination is in agreement with a conservation law obtained from equations (15). From the combination for the total momentum conservation it follows

$$\rho u = \rho'_1 u_1 + \rho'_2 u_2$$

and from (16)

$$u = c \, u_1 + (1 - c) \, u_2 \; . \tag{23}$$

Selection of parameters for the thermal impulse and specific entropy may follow from the characterisation of the total thermal and entropy fluxes

$$\rho'_1 q_1 + \rho'_2 q_2$$

$$\rho'_1 s_1 + \rho'_2 s_2$$

and alternative combinations follow from the fluxes' differences:

$$\rho'_1 q_1 - \rho'_2 q_2 \tag{24}$$

and

$$\rho'_1 s_1 - \rho'_2 s_2 \; . \tag{25}$$

A natural restriction follows from the energy conservation laws in (9) and (10), from which the total energy is calculated as follows

$$\rho'_1 (e_1 + u_1^2/2) + \rho'_2 (e_2 + u_2^2/2) \; . \tag{26}$$

Because specific energy is an extensive variable, the averaged specific energy is

$$e = c \, e_1 + (1 - c) \, e_2 \; . \tag{27}$$

Thus, it follows that (26) and (27) should give

$$\rho \left(E + \frac{u^2}{2} \right) = \rho'_1 \left(e_1 + \frac{u_1^2}{2} \right) + \rho'_2 \left(e_2 + \frac{u_2^2}{2} \right),$$

where u is determined from (23). Then, using (16), we have:

$$E + \frac{u^2}{2} = c\left(e_1 + \frac{u_1^2}{2}\right) + (1-c)\left(e_2 + \frac{u_2^2}{2}\right) = e + \frac{cu_1^2 + (1-c)u_2^2}{2} \ .$$

Therefore, using (23), we must define a generalised internal energy E as follows

$$E = e + \frac{cu_1^2 + (1-c)u_2^2 - (cu_1 + (1-c)u_2)^2}{2} = e + \frac{c(1-c)(u_1 - u_2)^2}{2} \ . \tag{28}$$

It should be noted that the definition (28) is not dependent on how a choice of variables affects the velocity difference. The only restrictions imposed were those on the total momentum (23) and on the averaged specific internal energy (27).

To choose a parameter for the velocity fluctuation we have two options (21) and (22). We start our analysis with the option (21), denoting a velocity difference parameter by w, so that

$$\rho w = \rho'_1 u_1 - \rho'_2 u_2$$

or, using (16) and (23), we have

$$u_1 = \frac{u + w}{2c}, \quad u_2 = \frac{u - w}{2(1-c)} . \tag{29}$$

This definition reduces the generalised internal energy (28) to the following

$$E = e + \frac{[(1-2c)u + w]^2}{4c(1-c)} \ .$$

Subtraction of the momentum equations from (9) and (10) along with (19) and (29) gives

$$\frac{\partial \rho w}{\partial t} + \frac{\partial\left\{\rho uw + \dfrac{u^2(1-2c) + (2-4c+4c^2)uw + w^2(1-2c)}{4c(1-c)}\right\}}{\partial x} + \frac{\partial[\theta p_1 - (1-\theta)p_2]}{\partial x} = 2n_0 \ .$$

Thus, this governing equation clearly shows that evolution of the parameter w is determined by the gradient of a function which can not be associated with a potential of the generalised internal energy E. For this reason this parametric choice was considered to be unsuitable for our purposes.

Thus, the only viable option for the selection of parameters characterising momentum, thermal impulse and entropy are combinations for the total momentum, total thermal impulse and total entropy flux complemented with the combinations (22), (24), and (25).

With the use of (16) for the thermal impulse and entropy flux, these combinations result in the following equations:

$$q = cq_1 + (1-c)q_2 , \quad \lambda = cq_1 - (1-c)q_2 ,$$
$$s = cs_1 + (1-c)s_2 , \quad \chi = cs_1 - (1-c)s_2 . \tag{30}$$

A generalisation for multi-phase (n-phase) case might take the following form for λ_i (an analogue of the parameter λ): $\lambda_i/n = c_i q_i - q/n$ (i=1,…,n). It is easy to see that for the two-phase case this definition is reduced to the definition (30): $\lambda = 2c\,q_1 - [c\,q_1 + (1-c)\,q_2]$ or $\lambda = c\,q_1 - (1-c)\,q_2$. Thus, the phase thermal impulse and entropy can be calculated, using the present choice (30) that results in

$$q_1 = \frac{q+\lambda}{2c} , \quad q_2 = \frac{q-\lambda}{2(1-c)} , \quad s_1 = \frac{s+\chi}{2c} , \quad s_2 = \frac{s-\chi}{2(1-c)} . \tag{31}$$

Using the definitions (31), (16) and (23), and the combination (22) that was selected for our derivation, we can write down definitions of parameters involving velocities:

$$u = cu_1 + (1-c)u_2 , \quad \rho v = u_1 - u_2 - \frac{\rho}{4}\left[\frac{(q+\lambda)(s+\chi)}{c} - \frac{(q-\lambda)(s-\chi)}{1-c} \right]. \tag{32}$$

For the sake of convenience we denote

$$v' = v + \frac{1}{4}\left[\frac{(q+\lambda)(s+\chi)}{c} - \frac{(q-\lambda)(s-\chi)}{1-c} \right], \tag{33}$$

then from (32) and (33) the phase velocities are

$$u_1 = u + \rho\,v'\,(1-c) , \quad u_2 = u - \rho\,v'\,c . \tag{34}$$

Summarising, the generalised energy can be calculated from (28) as follows

$$E = e + \frac{c(1-c)(\rho v')^2}{2} , \tag{35}$$

where v', which is defined by (33), is a function of v, q, s, c, λ, and χ. It should be noted that with the present choice of velocity parameters (32) the variable v does not have a dimension of velocity, similarly to the definition of velocity difference in the compact formulation of the model [1] obtained in [2].

4. Equation of state for the two-phase model

Using the equations of state (18) for the phases and definitions (17) and (31), we can build up the equation of state for an averaged medium:

$$e(\rho, c, \theta, q, \lambda, s, \chi) = c\, e_1(\rho c/\theta, (q + \lambda)/(2c), (s + \chi)/(2c)) +$$

$$+ (1 - c)\, e_2(\rho(1 - c)/(1 - \theta), (q - \lambda)/(2(1 - c)), (s - \chi)/(2(1 - c)))\ . \tag{36}$$

Summarising the chosen set of parameters, the kinematic variable of the model is averaged velocity u and internal variables (independent thermodynamic parameters) are ρ, c, θ, q, λ, s, χ, and v. Using (35) along with (36), we can calculate derivatives of E with respect to each of the independent thermodynamic parameters. We start with the derivative of E with respect to the mass concentration c:

$$E_c = e_1 - e_2 + c\frac{\partial e_1}{\partial \rho_1}\frac{\rho}{\theta} - (1-c)\frac{\partial e_2}{\partial \rho_2}\frac{\rho}{1-\theta} - c\frac{\partial e_1}{\partial q_1}\frac{q+\lambda}{2c^2} + (1-c)\frac{\partial e_2}{\partial q_2}\frac{q-\lambda}{(1-c)^2} -$$

$$- c\frac{\partial e_1}{\partial s_1}\frac{s+\chi}{2c^2} + (1-c)\frac{\partial e_2}{\partial s_2}\frac{s-\chi}{(1-c)^2} + \frac{(1-2c)(\rho v')^2}{2} + c(1-c)\rho^2 v'\frac{\partial v'}{\partial c} =$$

$$= e_1 - e_2 + \rho_1\frac{\partial e_1}{\partial \rho_1} - \rho_2\frac{\partial e_2}{\partial \rho_2} - q_1\frac{\partial e_1}{\partial q_1} + q_2\frac{\partial e_2}{\partial q_2} - s_1\frac{\partial e_1}{\partial s_1} + s_2\frac{\partial e_2}{\partial s_2} + \frac{(1-2c)(\rho v')^2}{2} -$$

$$- c(1-c)\frac{\rho^2 v'}{4}\left[\frac{(q+\lambda)(s+\chi)}{c^2} + \frac{(q-\lambda)(s-\chi)}{(1-c)^2}\right].$$

Combining the result with (17), (20), (31), (12) and (4), we obtain the derivative in a compact form, which may be called a generalized chemical potential:

$$\Lambda = E_c = \mu_1 - \mu_2 + \frac{(1-2c)(\rho v')^2}{2} -$$

$$- c(1-c)\frac{\rho^2 v'}{4}\left[\frac{(q+\lambda)(s+\chi)}{c^2} + \frac{(q-\lambda)(s-\chi)}{(1-c)^2}\right]. \tag{37}$$

It should be mentioned that a similar inverse dependence of phase entropies on the mass concentration chosen in [14] proved to be a suitable representation for the potential E_c to be associated with an affinity of the Gibbs potentials.

The derivative of E with respect to the specific density ρ can be associated with a generalised pressure P and it is calculated as follow

$$P = \rho^2 E_\rho = \rho^2 \left[c \frac{\partial e_1}{\partial \rho_1} \frac{c}{\theta} + (1-c) \frac{\partial e_2}{\partial \rho_2} \frac{1-c}{1-\theta} \right] + c(1-c)\rho^3 (v')^2 =$$
$$= \theta p_1 + (1-\theta) p_2 + c(1-c)\rho^3 (v')^2 . \tag{38}$$

The derivative of E with respect to θ can be calculated with the use of (17), (20) and (36):

$$\Pi = E_\theta = -c \frac{\partial e_1}{\partial \rho_1} \frac{\rho c}{\theta^2} + (1-c) \frac{\partial e_2}{\partial \rho_2} \frac{\rho(1-c)}{(1-\theta)^2} = -\frac{p_1 - p_2}{\rho} . \tag{39}$$

A generalised characteristic associated with the entropy flux J could be obtained by differentiation of E with respect to q:

$$J = E_q = c \frac{\partial e_1}{\partial q_1} \frac{1}{2c} + (1-c) \frac{\partial e_2}{\partial q_2} \frac{1}{2(1-c)} + c(1-c)\rho^2 v' \frac{\partial v'}{\partial q} =$$
$$= \frac{1}{2} J_1 + \frac{1}{2} J_2 + c(1-c) \frac{\rho^2 v'}{4} \left(\frac{s+\chi}{c} - \frac{s-\chi}{1-c} \right), \tag{40}$$

and, similarly, generalised temperature

$$T = E_s = c \frac{\partial e_1}{\partial s_1} \frac{1}{2c} + (1-c) \frac{\partial e_2}{\partial s_2} \frac{1}{2(1-c)} + c(1-c)\rho^2 v' \frac{\partial v'}{\partial s} =$$
$$= \frac{1}{2} T_1 + \frac{1}{2} T_2 + c(1-c) \frac{\rho^2 v'}{4} \left(\frac{q+\lambda}{c} - \frac{q-\lambda}{1-c} \right). \tag{41}$$

Derivatives with respect to λ and χ can be obtained in a similar way:

$$B = E_\lambda = c \frac{\partial e_1}{\partial q_1} \frac{1}{c} - (1-c) \frac{\partial e_2}{\partial q_2} \frac{1}{1-c} + c(1-c)\rho^2 v' \frac{\partial v'}{\partial \lambda} =$$
$$= \frac{1}{2} J_1 - \frac{1}{2} J_2 + c(1-c) \frac{\rho^2 v'}{4} \left(\frac{s+\chi}{c} + \frac{s-\chi}{1-c} \right), \tag{42}$$

and

$$A = E_\chi = c \frac{\partial e_1}{\partial s_1} \frac{1}{c} - (1-c) \frac{\partial e_2}{\partial s_2} \frac{1}{1-c} + c(1-c)\rho^2 v' \frac{\partial v'}{\partial \chi} =$$
$$= \frac{1}{2} T_1 - \frac{1}{2} T_2 + c(1-c) \frac{\rho^2 v'}{4} \left(\frac{q+\lambda}{c} + \frac{q-\lambda}{1-c} \right). \tag{43}$$

Temperatures and entropy fluxes of the individual phases can be calculated from (41), (43) and (40), (42), respectively, as follows

$$T_1 = T + A - c(1-c)\frac{\rho^2 v'}{2} \cdot \frac{q+\lambda}{c}, \quad T_2 = T - A + c(1-c)\frac{\rho^2 v'}{2} \cdot \frac{q-\lambda}{1-c}. \tag{44}$$

and

$$J_1 = J + B - c(1-c)\frac{\rho^2 v'}{2} \cdot \frac{s+\chi}{c}, \quad J_2 = J - B + c(1-c)\frac{\rho^2 v'}{2} \cdot \frac{s-\chi}{1-c}. \tag{45}$$

Finally, from (35) we need to calculate the derivative of E with respect to v:

$$\Omega = E_v = c(1-c)\rho^2 v' \frac{\partial v'}{\partial v} = c(1-c)\rho^2 v'. \tag{46}$$

This concludes the set of formulas for the determination of the dependent thermodynamical parameters of the model.

5. Conservation laws of the two-phase model

Conservation laws will be derived from the conservation laws (9) and (10) for partial parameters. We start with the mass conservation law. From (16) and (34): $\rho'_1 + \rho'_2 = \rho$ and $\rho'_1 u_1 + \rho'_2 u_2 = \rho u$. Using these relations and summing up the continuity equations in (9)-(10), we can obtain the continuity equation for the averaged variables:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0. \tag{47}$$

Rewriting the continuity equation in (9) with the use of (17) and (34), we can obtain the following governing equation for the mass concentration:

$$\frac{\partial \rho c}{\partial t} + \frac{\partial \left[\rho u c + \rho^2 c(1-c)v'\right]}{\partial x} = m_0. $$

From (46) this equation can be reduced to the following:

$$\frac{\partial \rho c}{\partial t} + \frac{\partial (\rho u c + \Omega)}{\partial x} = m_0, \tag{48}$$

where $\Omega = E_v$.

The compaction dynamics equation for the volume concentration is chosen in the well-know form of a conservation law within a material volume [3, 4, 6-9, 11, 14]:

$$\frac{\partial \rho \theta}{\partial t} + \frac{\partial \rho u \theta}{\partial x} = \Phi = -\frac{\rho \Pi}{\tau_0} \, ,$$ (49)

here Φ is a function responsible for the process of phase compaction and τ_0 is the relaxation time function governing the compaction kinetics. It should be mentioned that the kinetics chosen in (49) is quite natural because the compaction dynamics (rate of change of the volume fraction) should be associated with the pressure difference between the phases. This association can be easily seen from the definition (39) for Π.

The momentum conservation laws (the second equations of systems (9) and (10)) give the following momentum equation for the two-phase medium:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \left(\rho u^2 + P \right)}{\partial x} = 0 \, .$$ (50)

Here the following relationships have been used that were derived from equations (34) and (38):

$$\rho'_1 u_1 + \rho'_2 u_2 = \rho u \, ,$$

$$\rho'_1 (u_1)^2 + \rho'_2 (u_2)^2 + p = \rho u^2 + p + \rho^3 c(1 - c)(v')^2 = \rho u^2 + P \, .$$

As we discussed in Section 3, auxiliary conservation laws (15) for chemical flux will be used for the representation of the velocity difference between the phases. We subtract the second equation of (15) from the first one and for the first term of the left-hand side we use the representation (22) directly in the form (32). The second term of the left-hand side of the resultant equation can be calculated with the help of the following relationships:

$$(u_1)^2 - (u_2)^2 = (u_1 - u_2)(u_1 + u_2) = 2\rho u v' + (1 - 2c) \, (\rho v')^2 \, ,$$

$$\rho u v' = \rho u v + \frac{\rho u}{4} \left[\frac{(q + \lambda)(s + \chi)}{c} - \frac{(q - \lambda)(s - \chi)}{1 - c} \right] ,$$

and

$$\rho'_2 s_2 q_2 u_2 - \rho'_1 s_1 q_1 u_1 = \frac{\rho}{4} \left[\frac{(s - \chi)(q - \lambda)}{1 - c}(u - \rho v' c) - \frac{(s + \chi)(q + \lambda)}{c}(u + \rho v'(1 - c)) \right] =$$

$$= \frac{\rho u}{4} \left[\frac{(s - \chi)(q - \lambda)}{1 - c} - \frac{(s + \chi)(q + \lambda)}{c} \right] - c(1 - c) \frac{\rho^2 v'}{4} \left[\frac{(s - \chi)(q - \lambda)}{(1 - c)^2} + \frac{(s + \chi)(q + \lambda)}{c^2} \right] .$$

Using (37) and replacing corresponding parts in the second term by the results obtained above, we have the following expression for the derivand of the second term:

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$$\frac{u_1^2}{2} - \frac{u_2^2}{2} + \rho_2' s_2 q_2 u_2 - \rho_1' s_1 q_1 u_1 + \mu_1 - \mu_2 = \rho u v + \frac{\rho u}{4}\left[\frac{(q+\lambda)(s+\chi)}{c} - \frac{(q-\lambda)(s-\chi)}{1-c}\right] +$$

$$+ \frac{1-2c}{2}(\rho v')^2 + \frac{\rho u}{4}\left[\frac{(s-\chi)(q-\lambda)}{1-c} - \frac{(s+\chi)(q+\lambda)}{c}\right] -$$

$$- c(1-c)\frac{\rho^2 v'}{4}\left[\frac{(s-\chi)(q-\lambda)}{(1-c)^2} + \frac{(s+\chi)(q+\lambda)}{c^2}\right] + \mu_1 - \mu_2 = \rho u v + \mu_1 - \mu_2 +$$

$$+ \frac{1-2c}{2}(\rho v')^2 - c(1-c)\frac{\rho^2 v'}{4}\left[\frac{(s-\chi)(q-\lambda)}{(1-c)^2} + \frac{(s+\chi)(q+\lambda)}{c^2}\right] = \rho u v + E_c \ .$$

Thus, a conservation law for the chemical flux difference is:

$$\frac{\partial \rho v}{\partial t} + \frac{\partial(\rho u v + \Lambda)}{\partial x} = \Psi = N_1 - N_2 \ , \tag{51}$$

where Ψ is calculated as a difference of the right-hand sides in (15), using the parametric representations of the material-specific parameters (16), (31), and (34).

From (30) the total thermal impulse

$$\rho_1' q_1 + \rho_2' q_2 = \rho q$$

and after summation of the thermal impulse equations in (9) and (10) we have the following expression for derivand of the convective term with help of (16), (31), and (44):

$$\rho_1' q_1 u_1 + \rho_2' q_2 u_2 + T_1 + T_2 = \rho\left[\frac{(q+\lambda)}{2}(u + \rho v'(1-c)) + \frac{(q-\lambda)}{2}(u - \rho v' c)\right] +$$

$$+ 2T - \frac{\rho^2 v'}{2}[(q+\lambda)(1-c) - (q-\lambda)c] = \rho q u + 2T \ .$$

Thus, the conservation law for the thermal impulse takes the following form

$$\frac{\partial \rho q}{\partial t} + \frac{\partial(\rho u q + 2T)}{\partial x} = -F \ , \tag{52}$$

where

$$F = \frac{\rho}{2}\left(\frac{q+\lambda}{\tau_1} + \frac{q-\lambda}{\tau_2}\right) = \varphi_1 + \varphi_2 \ ,$$

and the arguments of the thermal relaxation times τ_1 and τ_2 are calculated with the use of the phase's parameter representations (17) and (34). It is interesting to note use of 2 as a

denominator for the flux of the generalised temperature in the conservation law (52) (for an n-phase mixture the denominator would be n, i.e. is the number of species in the mixture). The occurrence of this denominator in the present consideration results from an association of temperature and entropy flux with the total volume in contrast to partial characteristics such as density or pressure.

The conservation law for parameter λ can be obtained by subtraction of the corresponding equation for the thermal impulse in (10) from that in (9). The following relationship is used

$$\rho_1' q_1 u_1 - \rho_2' q_2 u_2 + T_1 - T_2 = \rho\left[\frac{(q+\lambda)}{2}(u + \rho v'(1-c)) - \frac{(q-\lambda)}{2}(u - \rho v'c)\right] +$$

$$+ 2E_\chi - \frac{\rho^2 v'}{2}\left[(q+\lambda)(1-c) + (q-\lambda)c\right] = \rho u \lambda + 2E_\chi \ .$$

Hence, the conservation law takes the form

$$\frac{\partial \rho \lambda}{\partial t} + \frac{\partial(\rho u \lambda + 2A)}{\partial x} = -G \ , \tag{53}$$

where A = E_χ and

$$G = \frac{\rho}{2}\left(\frac{q+\lambda}{\tau_1} - \frac{q-\lambda}{\tau_2}\right) = \varphi_1 - \varphi_2 \ .$$

Similarly, conservation laws for entropy s and parameter χ could be obtained from (14) and (45):

$$\frac{\partial \rho s}{\partial t} + \frac{\partial(\rho u s + 2J)}{\partial x} = R_1 + R_2 + \Delta \ , \tag{54}$$

and

$$\frac{\partial \rho \chi}{\partial t} + \frac{\partial(\rho u \chi + 2B)}{\partial x} = R_1 - R_2 = H \ , \tag{55}$$

where B = E_λ. The term Δ was added up in order to take into account dissipation that arises during compaction of components of the multi-phase mixture.

The energy conservation law could be derived directly from the conservation laws (47)-(55), the equation of state (35)-(36), and relationships for the thermodynamic dependent variables obtained in the previous Section (in fact, this procedure will be followed in the subsequent Section). Nevertheless, to confirm the consistency of the derivations we will derive the energy conservation law for the mixture directly from the phase conservation laws in (9)-(10). To derive it, we first obtain an auxiliary relationship:

$c\,(u_1)^2 + (1-c)\,(u_2)^2 = u^2 + \rho^2\,c(1-c)(v\,')^2$.

Using it, we can evaluate the following part of the energy flux:

$$
\rho_1' u_1\!\left(e_1 + \frac{u_1^2}{2}\right) + \rho_1' u_1\!\left(e_1 + \frac{u_1^2}{2}\right) + p_1' u_1 + p_2' u_2 =
$$

$$
= \rho\!\left[c\big(u + \rho v'(1-c)\big)\!\left(e_1 + \frac{u_1^2}{2}\right) + (1-c)\big(u - \rho v'c\big)\!\left(e_2 + \frac{u_2^2}{2}\right)\right] +
$$

$$
+ \big(u + \rho v'(1-c)\big)p_1' + \big(u - \rho v'c\big)p_2' = \rho u e + \rho^2 v'c(1-c)(e_1 - e_2) +
$$

$$
+ \frac{\rho u}{2}\big(cu_1^2 + (1-c)u_2^2\big) + \frac{\rho^2 v'c(1-c)}{2}\big(u_1^2 - u_2^2\big) + up + \rho^2 v'c(1-c)\!\left(\frac{p_1'}{\rho_1'} - \frac{p_2'}{\rho_2'}\right) =
$$

$$
= \rho u\!\left(E + \frac{u^2}{2}\right) + \rho c(1-c)(\rho v')^2\!\left(u + \frac{\rho v'(1-2c)}{2}\right) + \rho^2 v'c(1-c)\!\left(e_1 - e_2 + \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right).
$$

Using (44) and (45) along with definition (46), we can expand the expression $J_1 T_1 + J_2 T_2$ and evaluate the total energy flux as follows

$$
\rho u\!\left(E + \frac{u^2}{2}\right) + \Omega(\mu_1 - \mu_2) + pu + \rho c(1-c)(\rho v')^2\!\left(u + \frac{\rho v'(1-2c)}{2}\right) +
$$

$$
+ 2TJ + 2AB - \frac{\Omega^2}{4}\!\left(\frac{s + \chi}{c}\cdot\frac{q + \lambda}{c} + \frac{s - \chi}{1-c}\cdot\frac{q - \lambda}{1-c}\right).
$$

Using the definitions of chemical potential (37) and generalised pressure (38), we have the following compact expression for the energy flux

$$
\rho u\!\left(E + \frac{u^2}{2}\right) + \Omega\Lambda + Pu + 2TJ + 2AB .
$$

Thus, the energy conservation law takes the following form for the two-phase mixture:

$$
\frac{\partial\rho\big(E + u^2\!\big/2\big)}{\partial t} + \frac{\partial\big[\rho u\big(E + u^2\!\big/2\big) + Pu + \Omega\Lambda + Pu + 2TJ + 2AB\big]}{\partial x} = 0 . \tag{56}
$$

Summarizing, the complete system of equations for a two-phase medium involves the mass, momentum, and energy conservation laws

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \; ,$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \left(\rho u^2 + P \right)}{\partial x} = 0 \; ,$$

$$\frac{\partial \rho \left(E + {u^2}/{2} \right)}{\partial t} + \frac{\partial \left[\rho u \left(E + {u^2}/{2} \right) + Pu + \Lambda \Omega + 2TJ + 2AB \right]}{\partial x} = 0 \; ,$$

(57)

and conservation laws for mass and volume concentrations, the thermal impulse, and the fluctuations of the thermal impulse, entropy and chemical flux:

$$\frac{\partial \rho c}{\partial t} + \frac{\partial \left(\rho u c + \Omega \right)}{\partial x} = m_0 \; ,$$

$$\frac{\partial \rho \theta}{\partial t} + \frac{\partial \rho u \theta}{\partial x} = \Phi \; ,$$

$$\frac{\partial \rho q}{\partial t} + \frac{\partial \left(\rho u q + 2T \right)}{\partial x} = -F \; ,$$

$$\frac{\partial \rho \lambda}{\partial t} + \frac{\partial \left(\rho u \lambda + 2A \right)}{\partial x} = -G \; ,$$

(58)

$$\frac{\partial \rho \chi}{\partial t} + \frac{\partial \left(\rho u \chi + 2B \right)}{\partial x} = H \; ,$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \left(\rho u v + \Lambda \right)}{\partial x} = \Psi \; .$$

The system (57)-(58) is closed by the equation of state taken from (35)-(36) in the following form

$$E = E \left(\rho, c, \theta, q, \lambda, s, \chi, v \right) \; . \tag{59}$$

Therefore, the thermodynamic dependent variables can be calculated from the following generalised thermodynamic identity

$$T \, ds = dE - P \, d\rho / \rho^2 - \Lambda \, dc - A \, d\chi - B \, d\lambda - J \, dq - \Omega \, dv - \Pi \, d\theta \; . \tag{60}$$

Specifically, from (59) and (60) the closure relations for the dependent variables take the following form:

$$P = \rho^2 E_\rho \; , \quad \Lambda = E_c \; , \quad \Pi = E_\theta \; , \quad T = E_s \; , \quad A = E_\chi \; , \quad J = E_q \; , \quad B = E_\lambda \; , \quad \Omega = E_v \; . \tag{61}$$

The right-hand sides of the system (58) are determined by the internal mechanisms of the behaviour of the mixture.

It is seen that the jump conditions can be easily obtained for the model because the conservation laws (57)-(58) can be used directly for this purpose. In particular, from the

jump conditions it follows that the dependent thermodynamic parameters P, Λ, T, A, J, B, and Ω and the averaged velocity u are the parameters which are conserved through the contact jump (when ignoring the parameter evolution, associated with the interphase friction and internal relaxation determined by the right-hand sides of (58)).

6. Correctness of the model

The energy conservation law can also be obtained from the system (58)-(59) in which the energy conservation law has been replaced by the conservation law for entropy (54). Using this approach for the derivation, equations of the system are multiplied by appropriate coefficients and after summation they result in the energy conservation law. We will conduct a backward equivalent procedure, expanding the energy conservation law after which the entropy conservation law is obtained and conditions of correctness can be found.

Thus, expanding the equation (56) and using the mass conservation law, we can reduce it to the following

$$\rho \frac{dE}{dt} + \rho u \frac{du}{dt} + \frac{\partial Pu}{\partial x} + \frac{\partial \Omega \Lambda}{\partial x} + 2\frac{\partial TJ}{\partial x} + 2\frac{\partial AB}{\partial x} = 0 \, ,$$

here $d/dt = \partial/\partial t + u\,\partial/\partial x$ is the particle derivative. Expanding this formula with the use of the mass and momentum conservation laws, equations (58), and the relationships (61), we have

$$\Omega\left(\Psi - \frac{\partial \Lambda}{\partial x}\right) + \Lambda\left(m_0 - \frac{\partial \Omega}{\partial x}\right) + \Pi\Phi + J\left(-F - 2\frac{\partial T}{\partial x}\right) + B\left(-G - 2\frac{\partial A}{\partial x}\right) + \rho T \frac{ds}{dt} +$$

$$+ A\left(H - 2\frac{\partial B}{\partial x}\right) + \frac{\partial \Omega \Lambda}{\partial x} + 2\frac{\partial TJ}{\partial x} + 2\frac{\partial AB}{\partial x} = 0 \, .$$

Replacing the particle derivative of entropy from (54) and observing cross-annihilation of space derivatives, we can eventually obtain a restriction on the right-hand sides of the system (58) and equation (54):

$$\Omega\Psi + \Lambda m_0 + \Pi\Phi - JF - BG + T\left(R_1 + R_2 + \Delta\right) + AH = 0 \, .$$

Now, we can rewrite denotations of N_1 and N_2 in (15), when using $\Psi = N_1 - N_2$, through R_1 and R_2 in (14) as follows

$$N_1 = \frac{n_0 - u_1 m_0}{\rho c} - q_1 R_1 + q_1 s_1 m_0 + \varphi_1 s_1 \, , \quad N_2 = -\frac{n_0 - u_2 m_0}{\rho(1-c)} - q_2 R_2 - q_2 s_2 m_0 + \varphi_2 s_2 \, .$$

Expanding F, G, and H with the substitution of their expressions in (52), (53), and (55), we have

$$\Omega\Psi + \Lambda m_0 + \Pi\Phi - J(\varphi_1 + \varphi_2) - B(\varphi_1 - \varphi_2) + T(R_1 + R_2 + \Delta) + A(R_1 - R_2) = 0.$$

Using the relationships (37) and (46) for Λ and Ω and presenting the averaged variables back through the variables for each phase (material-specific variables) with the help of equations (44) and (45), from the last two formulas it follows

$$\Omega\left(\frac{n_0 - u_1 m_0}{\rho c} - q_1 R_1 + q_1 s_1 m_0 + \varphi_1 s_1 + \frac{n_0 - u_2 m_0}{\rho(1-c)} + q_2 R_2 + q_2 s_2 m_0 - \varphi_2 s_2\right) +$$

$$+ \left(\mu_1 - \mu_2 + \frac{(1-2c)(\rho v')^2}{2} - \Omega(q_1 s_1 + q_2 s_2)\right) m_0 + \Pi\Phi -$$

$$- J_1\varphi_1 - \Omega s_1\varphi_1 - J_2\varphi_2 + \Omega s_2\varphi_2 + T_1 R_1 + \Omega R_1 q_1 + T_2 R_2 - \Omega R_2 q_2 + T\Delta = 0.$$

After obvious simplifications we have

$$\rho v'c(1-c)\left(\frac{n_0 - u_1 m_0}{c} + \frac{n_0 - u_2 m_0}{1-c}\right) + m_0(1-2c)(\rho v')^2 + m_0\rho u v' - u_1 n_0 + u_2 n_0 +$$

$$+ \Pi\Phi + T\Delta = 0$$

that is finally reduced to

$$\Pi\Phi + T\Delta = 0.$$

Thus, fulfilment of the energy conservation law imposes the following restriction on the compaction dissipation term introduced in (55):

$$\Delta = -\frac{\Pi\Phi}{T} = \frac{\rho\Pi^2}{T\tau_0}. \tag{62}$$

Therefore, as soon as the exchange terms l_0, m_0, and n_0 in (9)-(10) are chosen in such a way that $R_1 + R_2 \geq 0$ in (14), then the total entropy production is non-negative from (62).

For an analysis of when the system (57)-(58) is hyperbolic, we will conduct it in the way performed in [15] for the single-temperature model. The eigenvalue analysis is fairly complex for the system (57)-(58) that can be written in the following matrix form

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = B, \tag{63}$$

where $U^{\mathrm{T}} = \{ \rho, c, \theta, u, v, q, \lambda, s, \chi \}$. The point of the approach used in [15] was to reformulate a final system written in the mixture variables into an equivalent form, employing material-specific variables for the purpose of the eigenvalue analysis. For the present case the latter set of variables can be assembled as the following vector:

$$U^{\mathrm{T}} = \{\theta, \rho_1, u_1, q_1, s_1, \rho_2, u_2, q_2, s_2\} . \tag{64}$$

The compaction dynamics equation (49) can be reduced to the following one:

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} = \frac{\Phi}{\rho} . \tag{65}$$

We remind that the final system (57)-(58) (which could be used in an alternative form with the energy conservation law from (57) replaced by the entropy equation (54)) is a result of equivalent proper combinations of the systems (9), (10), and equations (14) and (15). In order to manipulate with the 'real' densities for each phase we use the following relationships:

$$\rho'_1 = \theta_1 \rho_1 , \qquad \rho'_2 = \theta_2 \rho_2 ,$$

where denoted $\theta_1 = \theta$ and $\theta_2 = 1 - \theta$. Keeping in mind the following relationship for the averaged density $\rho = \theta_1 \rho_1 + \theta_2 \rho_2$, we can derive from the continuity equation (47) and from the equation (48) for $\rho c = \rho'_1 = \theta_1 \rho_1$ the following equations for the phase densities:

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 u_1}{\partial x} + \frac{\rho_1 (u_1 - u)}{\theta_1} \frac{\partial \theta}{\partial x} = \frac{1}{\theta_1} \left(m_0 - \frac{\rho_1 \Phi}{\rho} \right) = M_1 ,$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 u_2}{\partial x} - \frac{\rho_2 (u_2 - u)}{\theta_2} \frac{\partial \theta}{\partial x} = \frac{1}{\theta_2} \left(-m_0 + \frac{\rho_2 \Phi}{\rho} \right) = M_2 . \tag{66}$$

From the entropy conservation law (54) and the conservation law for the entropy disequilibrium χ we can obtain the material-specific entropy conservation laws similar to (14) with an additive of $\Delta/2$ to the right hand sides. From these equations we can derive auxiliary equations for $\theta_1 s_1$ and $\theta_2 s_2$ with the use of relationships $\rho'_1 s_1 = \rho_1 \theta_1 s_1$ and $\rho'_2 s_2 = \rho_2 \theta_2 s_2$ and equations (66):

$$\rho_1 \left(\frac{\partial \theta_1 s_1}{\partial t} + u_1 \frac{\partial \theta_1 s_1}{\partial x} \right) - \rho_1 s_1 (u_1 - u) \frac{\partial \theta}{\partial x} + \frac{\partial J_1}{\partial x} = R_1 + \frac{\Delta}{2} - \theta_1 s_1 M_1 ,$$

$$\rho_2 \left(\frac{\partial \theta_2 s_2}{\partial t} + u_2 \frac{\partial \theta_2 s_2}{\partial x} \right) + \rho_2 s_2 (u_2 - u) \frac{\partial \theta}{\partial x} + \frac{\partial J_2}{\partial x} = R_2 + \frac{\Delta}{2} - \theta_2 s_2 M_2 . \tag{67}$$

Similarly, from the conservation laws (52) and (53) for the thermal impulse q and its disequilibrium λ, we can obtain the conservation laws for the phase's thermal impulses that are identical to the thermal impulse conservation laws in (9) and (10). After some

manipulation for exclusion of the volume concentration they give us another pair of auxiliary equations for $\rho_1 q_1$ and $\rho_2 q_2$:

$$
\begin{aligned}
&\theta_1\left(\frac{\partial \rho_1 q_1}{\partial t}+\frac{\partial \rho_1 q_1 u_1}{\partial x}\right)+\rho_1 q_1\left(u_1-u\right)\frac{\partial \theta}{\partial x}+\frac{\partial T_1}{\partial x}=-\varphi_1-\frac{\rho_1 q_1 \Phi}{\rho}, \\
&\theta_2\left(\frac{\partial \rho_2 q_2}{\partial t}+\frac{\partial \rho_2 q_2 u_2}{\partial x}\right)-\rho_2 q_2\left(u_2-u\right)\frac{\partial \theta}{\partial x}+\frac{\partial T_2}{\partial x}=-\varphi_2+\frac{\rho_2 q_2 \Phi}{\rho}.
\end{aligned}
\tag{68}
$$

After multiplication of the first equation of (67) by q_1 and the first equation of (68) by s_1 followed by their summation, we can obtain an equation for $\rho_1 \theta_1 q_1 s_1$ and, similarly, for $\rho_2 \theta_2 q_2 s_2$:

$$
\begin{aligned}
&\frac{\partial \rho_1 \theta_1 q_1 s_1}{\partial t}+\frac{\partial \rho_1 u_1 \theta_1 q_1 s_1}{\partial x}+s_1\frac{\partial T_1}{\partial x}+q_1\frac{\partial J_1}{\partial x}= \\
&\qquad=q_1\left(R_1+\frac{\Delta}{2}-\theta_1 s_1 M_1\right)+s_1\left(-\varphi_1-\frac{\rho_1 q_1 \Phi}{\rho}\right)=H_1, \\
&\frac{\partial \rho_2 \theta_2 q_2 s_2}{\partial t}+\frac{\partial \rho_2 u_2 \theta_2 q_2 s_2}{\partial x}+s_2\frac{\partial T_2}{\partial x}+q_2\frac{\partial J_2}{\partial x}= \\
&\qquad=q_2\left(R_2+\frac{\Delta}{2}-\theta_2 s_2 M_2\right)+s_2\left(-\varphi_2+\frac{\rho_2 q_2 \Phi}{\rho}\right)=H_2.
\end{aligned}
\tag{69}
$$

Rewriting the conservation law (51) for the chemical flux ρv in its equivalent form for the material-specific parameters (as a difference of equations in (1)), and then adding it to the first equation from (69) followed by subtraction of the second equation of (69), we can obtain the equation for the velocity difference in the following form:

$$
\begin{aligned}
&\frac{\partial\left(u_1-u_2\right)}{\partial t}+\frac{\partial\left(u_1^2/2-u_2^2/2\right)}{\partial x}+\frac{\partial\left(\mu_1-\mu_2\right)}{\partial x}+s_1\frac{\partial T_1}{\partial x}+q_1\frac{\partial J_1}{\partial x}-s_2\frac{\partial T_2}{\partial x}-q_2\frac{\partial J_2}{\partial x}= \\
&\qquad=\Psi+H_1-H_2.
\end{aligned}
$$

Expanding the derivative of the chemical potentials according to (6), this equation can be reduced to the following:

$$
\frac{\partial u_1}{\partial t}-\frac{\partial u_2}{\partial t}+u_1\frac{\partial u_1}{\partial x}-u_2\frac{\partial u_2}{\partial x}+\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}-\frac{1}{\rho_2}\frac{\partial p_2}{\partial x}=\Psi+H_1-H_2.
\tag{70}
$$

The momentum conservation law (50) can be rewritten in the form of a sum of the momentum conservation laws from (9)-(10). Excluding the variables ρ_i and θ_i from the derivands of (50), we obtain the following equation:

$$\theta_1\rho_1\left(\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}\right)+\theta_2\rho_2\left(\frac{\partial u_2}{\partial t}+u_2\frac{\partial u_2}{\partial x}+\frac{1}{\rho_2}\frac{\partial p_2}{\partial x}\right)+\left(p_1-p_2\right)\frac{\partial\theta}{\partial x}=0.$$

Combining appropriately this equation with (70), we can obtain equations for the velocities of each phase:

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}+\left(p_1-p_2\right)\frac{\partial\theta}{\partial x}=\theta_2\rho_2\left(\Psi+H_1-H_2\right)=G_1,$$

$$\frac{\partial u_2}{\partial t}+u_2\frac{\partial u_2}{\partial x}+\frac{1}{\rho_2}\frac{\partial p_2}{\partial x}+\left(p_1-p_2\right)\frac{\partial\theta}{\partial x}=-\theta_1\rho_1\left(\Psi+H_1-H_2\right)=G_2.$$

(71)

Finally, from (67) and (68) we can derive equations for q_i and s_i:

$$\frac{\partial q_1}{\partial t}+u_1\frac{\partial q_1}{\partial x}+\frac{1}{\rho_1\theta_1}\frac{\partial T_1}{\partial x}=\frac{1}{\rho_1\theta_1}\left(-\varphi_1-\frac{\rho_1 q_1\Phi}{\rho}\right)-\frac{q_1 M_1}{\rho_1}=F_1,$$

$$\frac{\partial q_2}{\partial t}+u_2\frac{\partial q_2}{\partial x}+\frac{1}{\rho_2\theta_2}\frac{\partial T_2}{\partial x}=\frac{1}{\rho_2\theta_2}\left(-\varphi_2+\frac{\rho_2 q_2\Phi}{\rho}\right)-\frac{q_2 M_2}{\rho_2}=F_2,$$

$$\frac{\partial s_1}{\partial t}+u_1\frac{\partial s_1}{\partial x}+\frac{1}{\rho_1\theta_1}\frac{\partial J_1}{\partial x}=\frac{1}{\rho_1\theta_1}\left(R_1+\frac{\Delta}{2}-\theta_1 s_1 M_1\right)-\frac{s_1\Phi}{\rho\theta_1}=N_1,$$

$$\frac{\partial s_2}{\partial t}+u_2\frac{\partial s_2}{\partial x}+\frac{1}{\rho_2\theta_2}\frac{\partial J_2}{\partial x}=\frac{1}{\rho_2\theta_2}\left(R_2+\frac{\Delta}{2}-\theta_2 s_2 M_2\right)+\frac{s_2\Phi}{\rho\theta_2}=N_2.$$

(72)

Summarising, from (65), (66), (71), and (72) we can obtain a system of equations, which is equivalent to the system (57)-(58), in the following form:

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}=\frac{\Phi}{\rho},$$

$$\frac{\partial\rho_1}{\partial t}+u_1\frac{\partial\rho_1}{\partial x}+\rho_1\frac{\partial u_1}{\partial x}+a_1\frac{\partial\theta}{\partial x}=M_1,$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+\frac{1}{\rho_1}\frac{\partial p_1}{\partial x}+b\frac{\partial\theta}{\partial x}=G_1,$$

$$\frac{\partial q_1}{\partial t}+u_1\frac{\partial q_1}{\partial x}+\frac{1}{\rho_1\theta_1}\frac{\partial T_1}{\partial x}=F_1,$$

$$\frac{\partial s_1}{\partial t}+u_1\frac{\partial s_1}{\partial x}+\frac{1}{\rho_1\theta_1}\frac{\partial J_1}{\partial x}=N_1,$$

(73)

$$\frac{\partial \rho_2}{\partial t} + u_2 \frac{\partial \rho_2}{\partial x} + \rho_2 \frac{\partial u_2}{\partial x} - a_2 \frac{\partial \theta}{\partial x} = M_2 \, ,$$

$$\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} + \frac{1}{\rho_2} \frac{\partial p_2}{\partial x} + b \frac{\partial \theta}{\partial x} = G_2 \, ,$$

$$\frac{\partial q_2}{\partial t} + u_2 \frac{\partial q_2}{\partial x} + \frac{1}{\rho_2 \theta_2} \frac{\partial T_2}{\partial x} = F_2 \, ,$$

$$\frac{\partial s_2}{\partial t} + u_2 \frac{\partial s_2}{\partial x} + \frac{1}{\rho_2 \theta_2} \frac{\partial J_2}{\partial x} = N_2 \, ,$$

where $a_1 = \rho_1(u_1 - u)/\theta_1$ and $a_2 = \rho_2(u_2 - u)/\theta_2$, and $b = p_1 - p_2$. We assume that the equation of state for each phase has independent mechanical and thermal impulse parts as proposed in [12]:

$$e_1(\rho_1, s_1, q_1) = e_1^X(\rho_1, s_1) + e_1^T(q_1), \quad e_2(\rho_2, s_2, q_2) = e_2^X(\rho_2, s_2) + e_2^T(q_2).$$

We will use the following denotations (specified just for the first phase):

$$C_1^2 = \frac{\partial p_1}{\partial \rho_1} = \frac{\partial}{\partial \rho_1}\left[\rho_1^2 \frac{\partial e_1^X}{\partial \rho_1}\right], \quad \Theta_1 = \frac{\partial^2 e_1^X}{\partial s_1^2}, \quad \Gamma_1 = \frac{\partial^2 e_1^X}{\partial \rho_1 \partial s_1}, \quad \Xi_1 = \frac{\partial^2 e_1^T}{\partial q_1^2}. \tag{74}$$

Thus, a presentation of the equivalent system (73) in the matrix form (63) for the vector of variables (64) can be obtained, where the matrix A is:

$$A = \begin{pmatrix} u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 & u_1 & \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & \dfrac{C_1^2}{\rho_1} & u_1 & 0 & \rho_1\Gamma_1 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{\Gamma_1}{\rho_1\theta_1} & 0 & u_1 & \dfrac{\Theta_1}{\rho_1\theta_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{\Xi_1}{\rho_1\theta_1} & u_1 & 0 & 0 & 0 & 0 \\ -a_2 & 0 & 0 & 0 & 0 & u_2 & \rho_2 & 0 & 0 \\ b & 0 & 0 & 0 & 0 & \dfrac{C_2^2}{\rho_2} & u_2 & 0 & \rho_2\Gamma_2 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\Gamma_2}{\rho_2\theta_2} & 0 & u_2 & \dfrac{\Theta_2}{\rho_2\theta_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dfrac{\Xi_2}{\rho_2\theta_2} & u_2 \end{pmatrix}, \tag{75}$$

and the right hand side in (63) is

$B^T = \{\Phi/\rho, M_1, G_1, F_1, N_1, M_2, G_2, F_2, N_2\}$.

Eigenvalues η of this system can be found from the following characteristic equation

$$\det(A - \eta I) = 0 .\tag{76}$$

Due to the structure of the matrix A (75), the characteristic equation (76) can be decomposed into the following product:

$$\xi_0 \left[\left(\xi_1^2 - C_1^2\right) \cdot \left(\xi_1^2 - \frac{\Theta_1 \cdot \Xi_1}{(\rho_1 \theta_1)^2}\right) - \frac{\Gamma_1^2 \Xi_1}{(\theta_1)^2} \right] \cdot \left[\left(\xi_2^2 - C_2^2\right) \cdot \left(\xi_2^2 - \frac{\Theta_2^2 \Xi_2^2}{(\rho_2 \theta_2)^2}\right) - \frac{\Gamma_2^2 \Xi_2}{(\theta_2)^2} \right] = 0 ,\tag{77}$$

where $\xi_0 = u - \eta$, $\xi_1 = u_1 - \eta$, and $\xi_2 = u_2 - \eta$. The solution $\xi_0 = 0$ corresponds to the evolution of the volume concentration and two multipliers in (77) correspond to the characteristic equations for the first and second phases. Analysing one of them, the conditions of the existence of the roots are

$$\Xi_1 \geq 0 , \quad C_1^2 \cdot \frac{\Theta_1 \cdot \Xi_1}{(\rho_1 \theta_1)^2} \geq \frac{\Gamma_1^2 \cdot \Xi_1}{(\theta_1)^2} .$$

Recalling denotations (74), we can rewrite the conditions as follows

$$\frac{\partial^2 e_1^T}{\partial q_1^2} \geq 0 , \quad \frac{\partial}{\partial \rho_1}\left[\rho_1^2 \frac{\partial e_1^X}{\partial \rho_1} \right] \cdot \frac{\partial^2 e_1^X}{\partial s_1^2} \geq \rho_1^2 \left(\frac{\partial^2 e_1^X}{\partial \rho_1 \partial s_1} \right)^2 .$$

After introduction of the specific volume as $V_1 = 1/\rho_1$, the conditions take the following well-known form

$$\frac{\partial^2 e_1^T}{\partial q_1^2} \geq 0 , \quad \frac{\partial^2 e_1^X}{\partial V_1^2} \cdot \frac{\partial^2 e_1^X}{\partial s_1^2} \geq \left(\frac{\partial^2 e_1^X}{\partial V_1 \partial s_1} \right)^2 ,\tag{78}$$

which represents the traditional conditions of the equation of state convexity for e_1^X and e_1^T. The same restriction should obviously be imposed for e_2^X and e_2^T as well.

7. Discussion and conclusions

The system of equations (57)-(58) has been obtained in a compact form for a two-phase heat-conducting medium. It appears that a generalisation for the multi-phase case can be conducted in an obvious way similar to the generalisation conducted in [2]. The conditions of hyperbolicity of the system (78) have been reduced to the requirement of convexity for the mechanical and thermal parts of the equation of state for individual phases. It should

be noted that non-strict hyperbolicity might be associated with the phase degeneration and a degeneration of the heat conductivity (which may happen when $\Xi_i=0$ in (74)) that is out of the scope of the report. The correctness of the model has been linked with the requirement of non-negative dissipation (62) associated with the compaction dynamics. It is interesting to note that the structure of the characteristic equation (77) preserves the characteristic structure of the original single-phase model when expanding it to the two-phase case. This allows one to solve a Riemann problem for a two-phase mixture with the help of single-phase-model Riemann solvers for each phase, facilitating development of a suitable numerical algorithm based on the Godunov method.

It should be noted that the generalisation of the model to the multi-dimensional case does not look obvious, and the major reason is the difficulties in a tensorial generalisation of the equation of state.

The appearance of the denominator 2 (that presents the number of species in a two-phase mixture), entering the governing equations (54) for entropy, equations in the system (58) for the thermal impulse and for fluctuations of the thermal impulse and entropy, and the energy conservation law in (57) can be interpreted by the fact that the dependent thermodynamic parameters T and J are not really partial parameters, in contrast to density and pressure.

8. References

1. Romenski E., Resnyansky A.D., and Toro E.F., Conservative hyperbolic model for compressible two-phase flow with different phase pressures and temperatures, Newton Institute Preprint Series, Preprint NI04034-NPA, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, 2004.

2. Resnyansky A.D. A Thermodynamically Complete Model for Simulation of One-Dimensional Multi-Phase Flows, DSTO report, DSTO-TR-1510, Edinburgh, Australia, 2003.

3. Baer M.R. and Nunziato J.W., A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials, Int. J. Multiphase Flow, v. 12, n. 6, 1986, pp. 861-889.

4. Passman S.L., Nunziato J.W., and Walsh E.K., A Theory of Multiphase Mixtures, in Rational Thermodynamics, 2nd ed., Springer-Verlag, Berlin, 1984, Appendix 5c, pp. 286-325.

5. Truesdell C., Rational Thermodynamics, 1969, McGraw-Hill, NY.

6. Bdzil J.B., Menikoff R., Son S.F., Kapila A.K., and Stewart D.S., Two-phase modeling of deflagration-to-detonation transition in granular materials: A critical examination of modeling issues, Phys. Fluids, v. 11, n. 2, 1999, pp. 378-402.

7. Kapila A.K., Menikoff R., Bdzil J.B., Son S.F., and Stewart D.S., Two-phase modeling of deflagration-to-detonation transition in granular materials: Reduced equations, Phys. Fluids v. 13, n. 10, 2001, pp. 3002-3024.

8. Gavrilyuk S. and Saurel R., Mathematical and numerical modeling of two-phase compressible flows with micro-inertia, J. Comp. Physics, 2002, v. 175, n. 1, pp. 326-360.

9. Resnyansky A.D., Milton B.E., and Romensky E.I., A Two-Phase Shock-Wave Model of Hypervelocity Liquid Jet Injection into Air, JSME Centennial Grand Congress, Int. Conf. On Fluid Engnr., JSME ICFE-97-228, 1997, pp. 943-947.

10. Mitrofanov V.V., Detonation waves in heterogeneous systems [in Russian], Novosibirsk State University, Novosibirsk, Russia, 1988.

11. Romenski E., Zeidan D., Slaouti A., and Toro E.F., Hyperbolic Conservative Model for Compressible Two-Phase Flow, Reprint of the Isaac Newton Institute for Mathematical Sciences, NI03022-NPA, Cambridge, UK, 2003.

12. Romenskii E.I., Hyperbolic equations of Maxwell's nonlinear model of elastoplastic heat-conducting media, Siberian Math. J., v. 30, n. 4, 1989, pp. 606 - 625.

13. Müller I. And Ruggeri T., Rational extended thermodynamics, Springer-Verlag, New York, 1998.

14. Resnyansky A.D. and Bourne N.K., Shock-wave compression of a porous material, J. of Applied Physics, 2004, 15 February, v. 95, n. 4, pp. 1760 - 1769.

15. Romenski E. and Toro E. F., Compressible Two-Phase Flows: Two-Pressure Models and Numerical Methods, Newton Institute Preprint Series, Preprint NI04012-NPA, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, 2004.

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19. ABSTRACT This publication reports on a compact presentation of a one-dimensional model for the description of two-phase flows with heat exchange. This model was proposed earlier in the form of a system of equations, containing the heat conduction and entropy conservation equations for each phase. In this report the system is formulated in a form that is consistent with the formulation of the preceding report of the author. The present system of equations uniformly describes the evolution of averaged variables and interphase fluctuation parameters. The correctness of the model has been established for convex equations of state and for proper compaction dynamics kinetics.